Exercise 3: Moving Average Filter (FIR-Filter)

For a successful conduction of the Laboratory Exercise an accurate **preparation** at home is **mandatory**. Otherwise the **participation** in the lab will be **refused** and a repetition of the exercise is required!

Preparation of the exercise (homework)

- Study the given example *„Example Digital Lowpass Filter.pdf“* (see folder “*ReadMe*”) from Lab Exercise 1 again.
- Recapitulate the chapters concerning the convolution of DT signals from the lecture in detail. Especially consider the practical implementation of discrete-time systems with a finite impulse response (see lecture notes).

  In digital signal processing average filters are widely-used and often preferred because of their simple structure and stability. The impulse response of a general continuous-time Moving Average Filter is illustrated below (Figure 1).

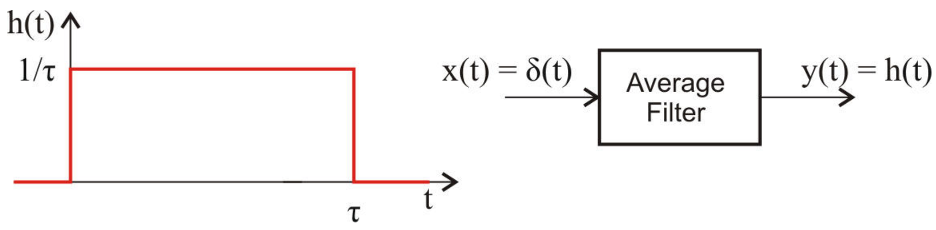

Figure 1: Impulse response of a continuous-time moving average filter

The relationship between the input and output of a moving average filter, for an arbitrary input is described by the following equation:

$$y(t) = \frac{1}{\tau} \cdot \int_{t-\tau}^{t} x(\lambda) d\lambda \qquad \tau: \text{ Averaging period}$$

- A typical application of the moving average filter is the reduction of interference from the power line (50 Hz) on small dc signals.

  Choose the averaging period τ of the filter so that the fundamental component and the harmonics are completely removed from the distorted signal.

  Which is the rise time of the filter due to a unit-step on the input?
  (Note: The rise time defines the required time for a signal to change from specified low value to specified high level. Typically, these values are 10% and 90% of the step height)

- Determine the Fourier transform of the system and draw the frequency response for the dedicated value τ (Frequency range f = 0 … 1 kHz, choose exceptionally a **linear scaling** for both axes).

  Which typical characteristic is associated with the filter?

- For the implementation of the digital filter the impulse response of the discrete-time average filter is defined as follows:

  $$h[n] = \begin{cases} \frac{1}{N}, & n = 0, 1, 2, \ldots, (N-1) \\ 0, & otherwise \end{cases}$$

  Determine N to reduce power line disturbances in case of a sampling rate F_S = 2 kHz.
  Using the impulse response of the DT system design the dedicated block diagram.

- The discrete-time moving average filter described previously is now **cascaded** with itself to improve the filter characteristic. In order to implement the **double average filter** as a single system, it is necessary to determine the resulting impulse response.

  Firstly draw the unit-impulse response of the cascaded moving average filter for **N = 4** (exceptionally for this part of exercise) using graphical convolution.

  How many samples does the impulse response contain?

Conduction of the exercise

Moving Average Filter (one stage)

- For the practical part of the laboratory exercise a powerful PEC80-Controller (from ABB Switzerland Ltd.) is provided again. The software for the PEC80-Controller can be developed in very comfortable way in the Matlab-Simulink by using a special toolbox (library).

  Please note that this library is only available for the **local installation** of **Matlab R2015a**.
  You can launch it directly with the desktop icon.

- Firstly verify the frequency response of the CT system with Matlab and print the result.
  Use the Fourier Transform from the preparation.

- For the implementation of the digital filter on the PEC80-Controller a Matlab-Simulink model needs to be build. As basis for your work a Matlab-Simulink template with the low-pass filter example is already provided (description see Lab 1 under "*ReadMe*").

  Now copy the Matlab-Simulink template (see folder "*Templates*") and unzip all files to your local Matlab workspace

  C:\Users\...\Projects\Matlab_2015a

- Then open the Simulink model "Implementation.mdl" which contains already the low-pass filter example. The model essentially consists of 3 different software tasks executed periodically on the controller later. The digital signal processing for the digital band-pass filter is done inside "Task A" whose cycle time is equal to the sample period. Whenever this task is called, the input signal x[n] (from "Analog In CH1") is read, the resulting output value y[n] of the filter (calculated by the function block " Digital Filter") is subsequently written to hardware (to "Analog Out CH1").

  Implement the block diagram (from the preparation) in Matlab / Simulink with x[n] as input and y[n] as output (Hint: you can use the existing diagram of the low-pass filter). For the Simulink model only the basic function blocks "Unit Delay", "Gain", Sum" can be used.

  Compile the Simulink model and download the generated code to your PEC80-Controller. (Note: check for the correct *IP address* of the controller). After the download is successfully completed the controller restarts itself automatically. This restart requires between 30 and 60 seconds and it is finished when the green R-Led is continuous on again.

- In the next step the frequency response of the single moving average filter is measured. Connect the provided function generator with dedicated analog input ("Analog In CH1") and display both the input and output signal on the oscilloscope. Determine the bode plot of the (magnitude and phase) of the filter by using the internal measurement functions of the oscilloscope between 10 Hz and 300 Hz. Choose exceptionally a **linear scaling** for frequency axis and the magnitude.

  How does the result match with the theoretical frequency response of a CT Moving Average Filter? Discuss possible reasons for the deviation.

- For analysis of the transient characteristic the step response of the moving average filter is examined. The function generator is well suited for providing a step input with an accurate edge. Configure the function generator with the suitable settings (Waveform, Amplitude, DC-Offset, etc.) so that the step has height of 1 V and duration of at least 1 second.

  Print the step response and measure the rise time.

Cascaded Moving Average Filter **(optional task, only if time permits)**

- Additionally the characteristic of the **double average filter** is analyzed.
  At first compute both the unit-pulse and unit-step response of the cascaded filter with the Matlab command *"conv"*. How many samples does the pulse response contain?
- For the implementation of the cascaded filter modify the existing block diagram in the Simulink Model, update the PEC80-Controller with the new software and measure the frequency response in the same way as before.
- The transient characteristic of the single and double average filter is compared now.
  Repeat the measure of the step response for the cascaded filter and identify the associated rise time.
- Compare the transient characteristic and frequency response of both average filters and describe the differences.

  Note that the convolution in time domain causes a multiplication in frequency domain.
  The related Fourier Transform Property is given by

  $$x(t) * h(t) \leftrightarrow X(\omega) \cdot H(\omega)$$

- Explain the alternative implementation of the double average filter as one single system.

- Additionally if the time permits, find the recursive implementation of single stage moving average filter and plot the frequency response in Matlab.

Save your data from the local workspace to an external drive (e. g. Pendrive) or your Home-Drive after each lab training!